ELASTICITY AND CREEP OF GLASS-REINFORCED PLASTIC 27-63s AT ELEVATED TEMPERATURES

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Several investigations of the elasticity and creep of oriented glass-reinforced plastics [1–4] revealed a considerable degree of anisotropy in these materials which are much more susceptible to shear creep than they are to creep in the direction of the reinforcing glass fibers. The equations used in [5, 6] to describe the behavior of oriented glass-reinforced plastics take into account creep in the direction of reinforcing fibers but only integrally, i.e., as an addition to the elastic strains; as a result, a real material is replaced by a hypothetical body which is subject only to shear creep.

Such an approach is convenient for selecting the optimum reinforcing parameters and for solving problems in which the tangential stresses are approximately the same as, or larger than, the normal stresses on the principal anisotropy planes.

In this connection, in many cases it seems desirable to make the mathematical model of such materials more precise and to take into account the nature of creep in the direction of the reinforcing fibers; this is especially important at elevated temperatures.

This article describes the results of tests in which glass-reinforced plastic 27-63s was strained in tension along one of the principal directions of anisotropy. The experimental data were processed to reveal the fundamental laws of elasticity and creep of the material in question.

The material—unwoven glass-reinforced plastic 26–63s—was supplied as a single batch in the form of plates measuring 250 × 250 × 3 mm. Each plate consisted of six unidirectional lamellae stacked in such a way that the glass fibers in adjacent lamellae were normal to each other; as a result, the material had equal strength in these two directions. The specimens were cut in the form of strips along one of the directions of the reinforcing fibers; they were 10 mm wide, 3 mm thick, and 250 mm long. Examination of the ground side-faces of the specimens showed that the longitudinal layers of glass fibers were not strictly rectilinear. The nonlinearity was of two kinds: either in the form of slight distortions (observed in all the specimens) or in the form which resembled corrugations and which was observed only in isolated portions of some of the specimens.

The first tests at elevated temperatures showed that specimens with corrugated glass fiber layers are characterized by stepwise creep and that their total strain is 50–100% larger than that recorded for specimens with weakly distorted glass fiber layers. Exfoliation of the former specimens along the interfaces between adjacent lamellae was observed after tests.

Subsequently, specimens of this kind were discarded; test results obtained for such specimens were not included in the analysis.

The tests were carried out on an apparatus described in [7]; the specimen temperature was maintained constant accurate to ±2° C, and the strains were measured with gages (with a scale value of 0.001 mm) on a specimen with a gage portion 100 mm long. A mounted specimen was heated to the test temperature in 1–2 hr and held at that temperature for 1/2 hr, after which a tensile load was applied in two stages (in 2–4 secs). Subsequently the load remained constant for 10 hr in the case of most specimens; in some cases, creep tests lasted several times longer. The test temperature ranged from 25 to 200° C. The applied stress of 10 kgf/mm^2 was chosen so as to investigate the variation in the elasticity and creep properties for the entire temperature interval while at the same stress level. It was found that this stress approaches the long-term strength of the material at 200° C.

The two-stage loading made it possible to plot three points on the graph of the relation $\sigma(\varepsilon)$. It was found that this relation (for $\sigma \leq 10$ kgf/mm^2) may be regarded as linear, since the ratios $\Delta\sigma/\Delta\varepsilon$ determined for the two stages were practically the same; the deviations were of opposite signs and random in nature.

The final calculations for the experimental values of the elasticity moduli were carried out with data relating to the full load: $E_{exp} = \sigma/\varepsilon$.

The test results are given below. Test temperatures are shown in the first line and elasticity moduli E_{exp} in the second line: their maximum and minimum values calculated for various specimens (top), the arithmetic mean values (bottom), and the number of specimens tested at a given temperature (in parentheses).

T° C =	25	40	70	100	150	200
E_{exp}, kgf/mm^2 =	2460—2860	2300—2810	2390—2690	1950—2630	1830—1960	1670—2000
	(3) 2650	(4) 2570	(3) 2560	(5) 2280	(2) 1900	(2) 1840
E_{cal}, kgf/mm^2 =	2653	2579	2431	2283	2036	1783
δ_E, % =	+0.1	+0.4	−5.3	+0.2	+6.7	−3.2

Glass fibers, of the kind used in structural materials, at temperatures of up to 200° C, mainly experience elasticity deformation; their time-dependent strains constitute a negligible part of the instantaneous strains. The temperature dependence of the elasticity modulus of glass fiber is linear [8].

If a specimen of a glass-reinforced plastic is cut along the direction of reinforcing fibers, the decisive part in its elasticity properties is played by longitudinal glass fibers. It is therefore natural to try to approximate the temperature dependence of the elasticity modulus of a glass-reinforced plastic by a linear relation.

The experimental data were processed by the method of least squares under the assumption of a linear correlation between E and T; as a result, the following formula was obtained:

$$E = 2776 - 4.93\,T \quad (E \text{ in kgf/mm}^2;\ T \text{ in } ^\circ\text{C}). \qquad (1)$$

Values of the elasticity moduli calculated from (1) and denoted by E_{cal} are given in the third line of the table above. While the relative deviations of E_{exp} (for various specimens) from the average value at a given temperature reach 13%, the relative difference between the average experimental and calculated values,

$$\delta_E \equiv \frac{E_{cal} - E_{exp}}{E_{cal}}, \qquad (2)$$

does not exceed 7% (see the fourth line in the table), i.e., formula (1) is applicable.

Creep test results are characterized by a wide scatter and creep curves obtained for different specimens tested at the same temperatures and stresses often differ not only quantitatively but also qualitatively.

This may be attributed to the existence of several mechanisms of creep and to the nonuniformity of the material, as a result of which various mechanisms may predominate in various specimens.

If it is assumed that glass fibers are rectilinear, a specimen of the kind under consideration can be modelled by two parallel elements: one which is elastic (and corresponds to the longitudinal glass fibers) and the other which is susceptible to creep (and corresponds to the transverse material layers and to the interlayers of the bonding constituent in the longitudinal layers). A model of this kind was used in [9–10] for the case of instantaneous deformations. Let the total strains of both elements be the same and let the strain-stress relationship for the first and second element be described by

$$\varepsilon = \frac{\sigma_1}{E_1},$$

$$\varepsilon = \frac{1}{E_2}\left[\sigma_2 + \chi \int_0^t \frac{(t-\tau)^\alpha}{\Gamma(1+\alpha)}\,\sigma_2(\tau)\,d\tau\right] \equiv \frac{1}{E_2}\left(1 + \chi J_\alpha^*\right)\sigma_2. \qquad (3)$$

Here σ_1, σ_2, E_1, and E_2 denote stresses and the elasticity moduli of the elements, Γ_α^* is the creep operator (see [11]), t is the time, and χ and α are constants. At σ_2 = const, it follows from (3) that

$$\varepsilon = \frac{\sigma_2}{E_2} + At^{1+\alpha} \quad (A = \text{const}).$$

If the relative cross-sectional areas of the elements are f_1 and f_2, respectively, we may write (for the average stress of the model)

$$\sigma = f_1\sigma_1 + f_2\sigma_2. \tag{4}$$

Equations (3) and (4) make it possible to describe the relation $\sigma(\varepsilon)$, for the model, in the form

$$\varepsilon = \lambda\,[1 + (\varkappa - \beta)\,\vartheta_\alpha^*(-\beta)]\,\sigma$$

$$\left(\beta = \frac{\chi f_1 E_1}{f_1E_1 + f_2E_2} > 0, \quad \lambda = \frac{3}{\chi f_1 E_1}\right). \tag{5}$$

Here $\vartheta_\alpha^*(-\beta)$ is Rabotnov's operator [11].

In accordance with the properties of this operator, creep at constant stress asymptotically diminishes. If $t \to \infty$, in formula (5),

$$\varepsilon \to \frac{\sigma}{E_1 f_1} \equiv \varepsilon_1^\infty.$$

Taking typical values of the parameters in question (E_1 = 7000 kgf/mm^2, f_1 = 0.3, f_2 = 0.7, and σ = 10 kgf/mm^2), we obtain ε_1^∞ = = 0.005. This limiting strain corresponds to the stage at which the entire load is carried by the elasticity element, i.e., the fiberglass.

The applicability of formula (5) for describing the behavior of real materials is illustrated by data in Fig. 1, where creep strain is plotted against time in logarithmic coordinates. The experimental points were obtained for one of the specimens tested (σ = 5 kgf/mm^2 and T = 100° C), while the curve corresponds to the function $4.1\,\vartheta_{-0.7}^*(-0.253)\,10^{-5}$. Unfortunately, these functions (or rather the Mittag-Leffler functions linearly related to them) are tabulated [12] only for values $\alpha = -0.7$, while most experimental curves obtained in the course of this investigation are characterized by larger values of α.

In addition to creep due to the redistribution of stress, deformation due to the straightening of glass fibers is possible. The form of the distorted longitudinal layers in specimens after testing resembles a sinusoid. Denoting the ratio of the sinusoid amplitude to the specimen length by $a/l = \rho$ and computing the length of the sinusoid arc $y = a\sin \pi x/l$

$$S = \frac{2l}{\pi}\sqrt{1 + (\pi\rho)^2}\,E\,(k, \tfrac{1}{2}\pi) \qquad \left(k = \frac{\pi\rho}{\sqrt{1 + (\pi\rho)^2}}\right), \tag{6}$$

where $E(k, \pi/2)$ is the complete elliptical integral of the second kind, we obtain an expression for the maximum strain of the material due to the straightening of glass fibers

$$\varepsilon_2^\infty = \frac{S - l}{l}. \tag{7}$$

For weakly distorted fibers, $\rho \geq 0.07$ of the factor in (6)

$$E\,(k, \tfrac{1}{2}\pi) \approx \pi/2 \quad \text{(see [13])}; \quad \sqrt{1 + (\pi\rho)^2} \approx 1 + \tfrac{1}{2}(\pi\rho)^2.$$

Then, from (7) we have

$$\varepsilon_2^\infty \approx 5\,\rho^2. \tag{8}$$

This formula makes it possible to estimate the maximum strain of the material due to the straightening of fibers in tension accurate to about 1% of ε_2^∞.

If the strain ε_2^∞ = 0.0002 (which is 4–5% of the elasticity strain at σ = 10 kgf/mm^2) is regarded as negligibly small, it follows from (8) that to obtain a material whose fibers may be regarded as straight, one must stipulate that ρ = 0.02.

The fact that ρ varies considerably from specimen to specimen is partly responsible for the qualitative and quantitative differences in the creep properties of these specimens, since termination of the mechanism in question for low ρ should be reflected in the nature of creep curves.

Creep of the material studied at room temperature (25° C) is accurately described by a power function in the entire time interval investigated (about 500 hr): when plotted in log ε^c-log t coordinates (where ε^c is creep strain), the experimental points obtained for various specimens tested at σ = 10 kgf/mm^2 lie on straight lines practically parallel to each other (Fig. 2).

This no longer holds at elevated temperatures (for which the duration of creep tests was 10 hr): under these circumstances, straight log ε^c-log t curves are obtained for some of the specimens, while the relation $\varepsilon^c(t)$ characterizing other specimens is nonlinear; in the latter case, the angle of the slope of the log ε^c-log t curves decreases with time as shown in Fig. 3, where data obtained at T = 100° C and σ = = 10 kgf/mm^2 are plotted. In both cases, it is possible, by chosing parameters α and β, to describe the experimental curves with the aid of the function $\vartheta_\alpha^*(-\beta)$; at β = 0 we obtain a power function

$$\varepsilon^c = At^{1+\alpha}. \tag{9}$$

The above expression was used to describe creep curves obtained at various temperatures. Values of the $(1 + \alpha)$ coefficient for specimens whose creep obeyed Eq. (9) without substantial deviations (at least in the 10–600 min time interval) are given below:

T, °C =	25	40	70	100	150
$1 + \alpha$ =	0.09—0.11 (5) 0.10	0.07—0.12 (2) 0.09	0.11—0.12 (2) 0.12	0.10—0.12 (3) 0.11	0.14—0.22 (2) 0.18

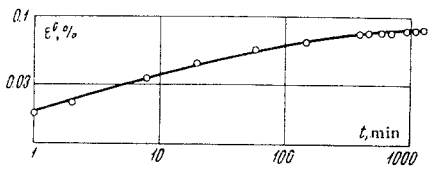

Fig. 1

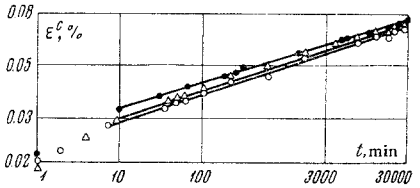

Fig. 2

Bearing in mind the considerable scatter of strain values recorded for replicate specimens, it may be conditionally agreed that all the remaining curves obey Eq. (9); the error will be 2–4% of the total strain and will be within the scatter band.

The creep at 200° C also followed Eq. (9), but a stepwise increase in strain at certain instants was observed in almost all the specimens; consequently, no data on $(1 + \alpha)$ at this temperature are reproduced above. The degree of scatter at 200° C is substantially higher and the creep strain at this temperature is several times larger than that at the lower temperatures.

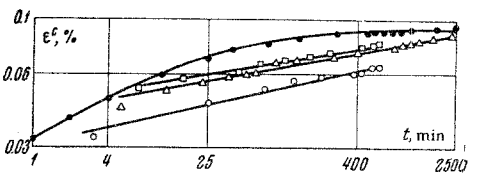

Fig. 3

Analysis of the results obtained indicates that the $(1 + \alpha)$ coefficient is practically independent of temperature in the interval 25–100° C, i.e., that the creep curves in this temperature interval are similar. Having selected an arbitrary instant (in this case $t_0 = 420$ min) and having determined $\varepsilon^c(t_0)$ for various specimens and temperatures, we can—utilizing the similarity of creep curves—fully describe the relationship between creep strain and temperature. Data obtained in this way were processed by the method of least squares; under the assumption of linear correlation between ε^c and T, we obtained

$$\varepsilon^c = (45 + 0.294\ T)\ 10^{-5}. \qquad (10)$$

The maximum relative deviation δ_ε of the average experimental strain from the calculated value is 1.0%:

T, °C =	25	40	70	100
$10^5\varepsilon^c_\vartheta$ =	44—63 / (5) 52	52—65 / (4) 57	60—70 / (2) 65	60—87 / (5) 74
$10^5\varepsilon^c_p$ =	52.4	56.8	65.6	74.4
δ_ε, % =	+0.8	−0.4	+1	+0.6 .

Tests on three specimens at $\sigma = 5$ kgf/mm^2 and T = 100° C showed that the average total strain is half that recorded for specimens tested at $\sigma = 10$ kgf/mm^2 and T = 100° C; consequently, it may be concluded that the stress-strain relation is linear.

Combining the results obtained, we may (at least for $\sigma \leq 10$ kgf/mm^2 and T ≤ 100° C) write an expression,

$$\varepsilon = \frac{\sigma}{a - bT} + 10^{-5}\frac{\sigma}{\sigma_0}(d + cT)\left(\frac{t}{t_0}\right)^{1+\alpha} + \gamma(T - 25).$$

Here $a = 2776$ kgf/mm^2; $b = 4.93$ kgf/mm^2; $\sigma_0 = 10$ kgf/mm^2; $1 + \alpha = 0.1$; $c = 0.2841$ °C^{-1}; $d = 45$; $t_0 = 420$ min; γ is the linear expansion coefficient (which, according to tentative determination, is equal to $1.3 \cdot 10^{-3}$ °C^{-1}.

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